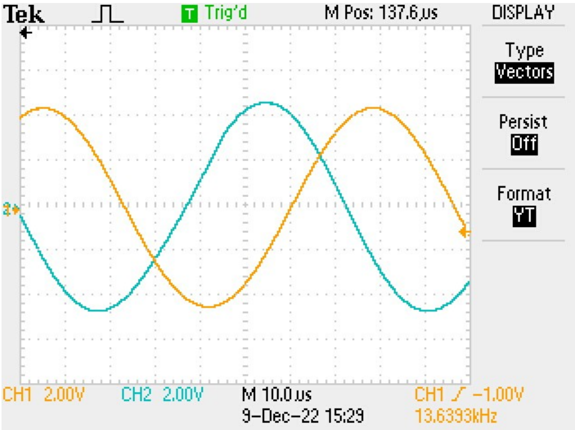
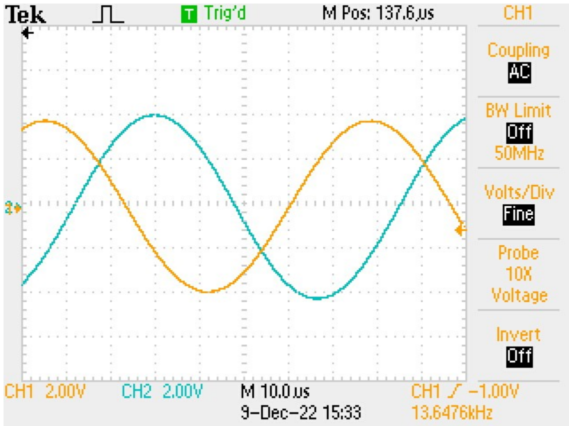
Supplement A

Test Results of the three phase sine wave generator

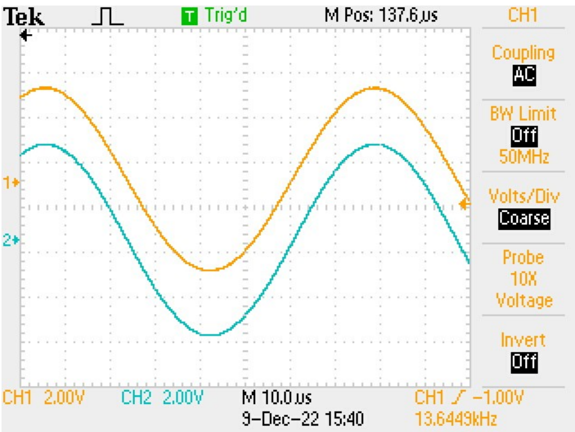
TP1 – yellow $V_m\sin(\omega t)$
TP2 – cyan $V_m\sin(\omega t+120°)$



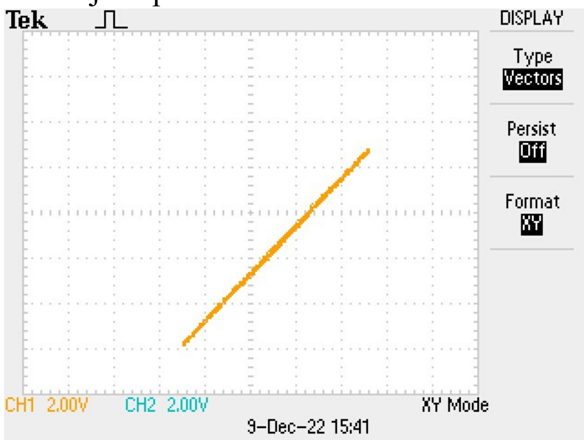
TP1 – yellow
TP3 – cyan $V_m\sin(\omega t+240°)$



TP1 – yellow
TP4 – cyan $V_m\sin(\omega t+360°)$



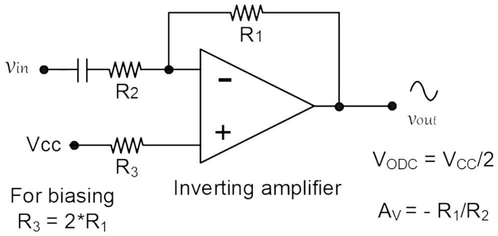
TP1 & TP4
Lissasjous plot



Theory and Analysis

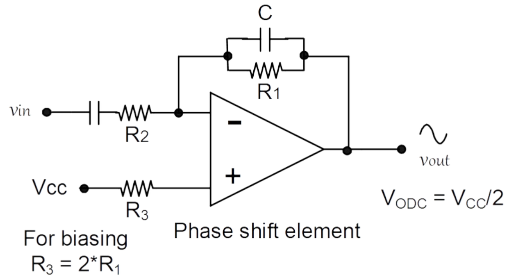
Phase Shifter

Figure 1



This is a basic inverting op-amp. For a sine wave signal output leads input by180°
Note: Input capacitor has low impedance at the frequencies considered and is ignored here.

Figure 2



Analysis with R//C feedback impedance Z

$$Z = \frac{\frac{R_1}{j\omega C}}{R_1 + \frac{1}{j\omega C}} = \frac{R_1}{j\omega R_1 C + 1}$$

$$X_C = \frac{1}{\varpi C}$$

$$Z = \frac{R_1}{j\frac{R_1}{X_c} + 1} = \frac{R_1\left(1 - j\frac{R_1}{X_c}\right)}{1 + \frac{R_1^{\,2}}{X_c^{\,2}}}$$

Since we are looking for 60 degree phase shift and tan 60° = $\sqrt{3}$, let $\frac{R_1}{X_c} = \sqrt{3}$

Then:

$X_C = \frac{R_1}{\sqrt{3}}$, substituting this into the equation for *Z* above:

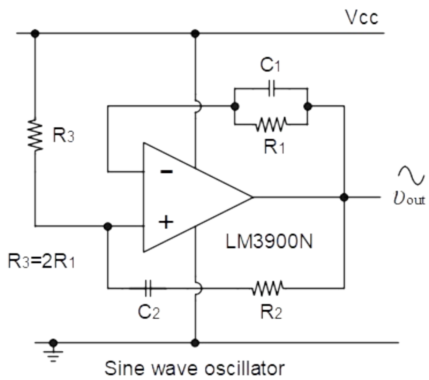
$$Z = \frac{R_1(1 - j\sqrt{3})}{4} = \frac{R_1}{2}\,\underline{/\text{-}60°}$$

Gain $A_V = (Z/R_2)$
Phase shifter needs unity gain therefore $R_1 = 2R_2$
$A_V = 1\underline{/\text{-}60°}$
Phase of the output voltage is
180°-60°=120°
$V_{in} = V_{out}$

Sine Wave Oscillator

Once the operating frequency for the three phase generator has been specified or calculated, sine wave source can be designed. Basic configuration is shown below:

Figure 3



Frequency of oscillation

$$\omega = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \text{ r/s} \qquad f = \frac{\omega}{2\pi} \text{ Hertz}$$

Condition for oscillation
$R_2/R_1 + C_1/C_2 = 1$

This is one of three oscillators described in October 2019 issue of Silicon Chip. For further details see free PDF file at: siliconchip.com.au/Shop/6/5073

Phase Shift Element

A project I have in mind needs a three phase sine wave source with the frequency in the range of 10 to 15 kHz. I picked a standard capacitor value of 10 nF. Some mathematics showed that a 20 kΩ resistor would get me in the range, thus:

resistor R_1 = 20 kΩ

$$\phi = \tan^{-1}\frac{R_1}{X_C}$$

phase shift $\phi = 60°$, $\tan\phi = \sqrt{3}$

$X_C = R_1/\sqrt{3} = 11547\Omega$

Also $X_C = 1/\omega C = 1/2\pi fC$

$f = 1/(2\pi * 11547 * 10 * 10^{-9}) = 13783$ Hz

For unity gain R_2=10 kΩ (R_1=2*R_2)

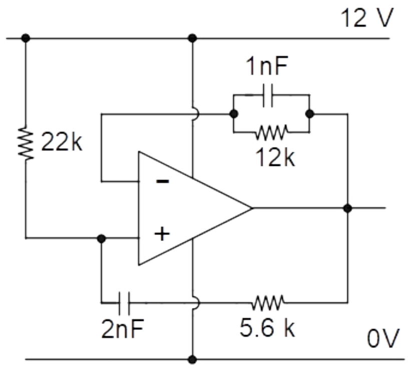
Sine Wave Oscillator

This oscillator (see Figure 3) is used as the sine wave source for the three phase generator.

Operating frequency=13783 Hz as defined by the phase shifter

Selecting C_1 as 1 nF and C_2 as 2 nF requires R_1 to be 11.55 kΩ and R_2 to be 5.77 kΩ

Actual circuit used:



Sine wave oscillator

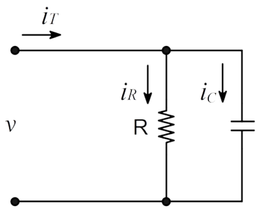
Standard resistor values were used. Measured frequency is 13.647 kHz.

Supplement B

Phase Shift Fundamentals

Analysis of Resistor and Capacitor Parallel Circuit

Figure 1



$$v = V_m \sin \omega t$$

$$i_T = i_R + i_C = \frac{v}{R} + C\frac{dv}{dt} = \frac{V_m}{R}\sin \omega t + \omega C V_m$$

$$X_C = \frac{1}{\varpi C}$$

Then $i_T = \sqrt{(1/R)^2 + (1/X_C)^2} * V_m \sin(\omega t + \tan^{-1}(R/X_C))$

The current leads the voltage by the angle $\phi = \tan^{-1}(R/X_C)$

Considering the special case where $\frac{R}{X_C} = \sqrt{3}$, (tan 60°= $\sqrt{3}$)

$X_C = R/\sqrt{3}$, then the term under the square root sign:

$\sqrt{(1/R)^2 + (1/X_C)^2} = \sqrt{(1/R)^2 + (\sqrt{3}/R)^2} = 2/R$

$$i_T = \frac{2V_m}{R}\sin(\omega t + \tan^{-1}\frac{R}{X_C}) = \frac{2V_m}{R}\sin(\omega t + 60°)$$

Operational amplifier feedback current

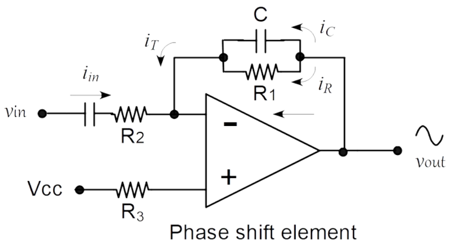
Norton operational amplifier is a current driven device and the generated output voltage of the amplifier will cause the following condition to be satisfied:

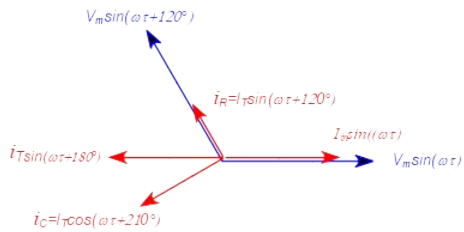
Input current = - Feedback current (in phase and magnitude)

In the circuit below the input voltage to the RC feedback circuit is the amplifier output voltage i.e.

$$v = V_O \sin(\omega t + 180 - \tan^{-1}\frac{R_1}{X_C})$$

Current in the RC circuit leads the voltage by $\tan^{-1}\frac{R_1}{X_C}$





$$i_{in} = \frac{V_m}{R_2}$$

$i_{in} = i_T$ (op-amp function)

$i_{R1} = \frac{v_{out}}{R_1}$, also from the phasor diagram:

$$i_{R1} = i_T \cos 60 = 0.5 i_T$$

if

$$R_1 = 2R_2$$

then

$$V_{in} = V_{out}$$

$$i_c = \frac{v_o}{X_c} = \sin 60$$

$$\frac{i_{R1}}{i_c} = \frac{R_1}{X_c} = \frac{\cos 60}{\sin 60} = \tan 60 = \sqrt{3}$$