# Analysis of Sinewave Circuits

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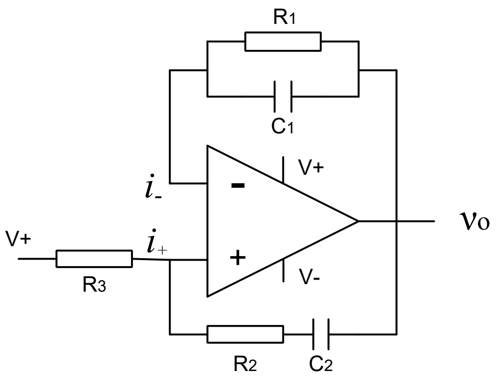
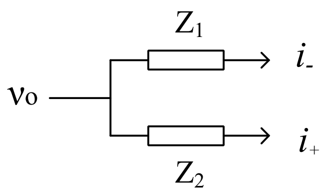
Three sinewave oscillator circuits based on Norton operational amplifier are analysed here using mesh current method. Mathematics of Circuit 1 is quite simple, involving only two currents. Circuits 2 and 3 have an extra loop current. See reference at the end of this document.

**Circuit 1**




$R_3$ is a bias resistor. $R_3 = 2 \bullet R_1$ therefore dc value of $|v_o| = ½ V+)$

**Circuit analysis:**

This simplified analysis of Circuit 1 allows one to select circuit elements to meet the condition for oscillation and determine the resulting frequency of oscillation.

$$Z_1 = \frac{\dfrac{R_1}{j\omega C_1}}{R_1 + \dfrac{1}{j\omega C_1}} = \frac{R_1}{j\omega R_1 C_1 + 1}$$

$$Z_2 = R_2 + \frac{1}{j\omega C_2}$$

For sustained oscillation:

$i_+ = i_-$ therefore $Z_1 = Z_2$

$$\frac{R_1}{j\omega R_1 C_1 + 1} = R_2 + \frac{1}{j\omega C_2}$$

$$\frac{j\omega R_1 C_2}{j\omega R_1 C_1 + 1} = j\omega R_2 C_2 + 1$$

$$j\omega R_1 C_2 = -\omega^2 R_1 C_1 R_2 C_2 + j\omega R_1 C_1 + j\omega RC + 1$$

Equate real and imaginary parts:

**REAL (**gives frequency of oscillation)**:**

$$\omega^2 R_1 C_1 R_2 C_2 = 1$$

$$\omega^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

$$\omega = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \text{ r/s}$$

$$f = \frac{\omega}{2\pi} \text{ Hertz}$$

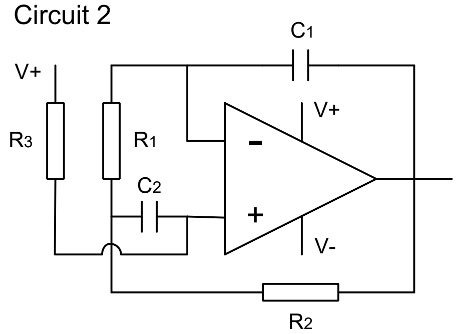
**IMAGINARY** (gives the condition for oscillation):

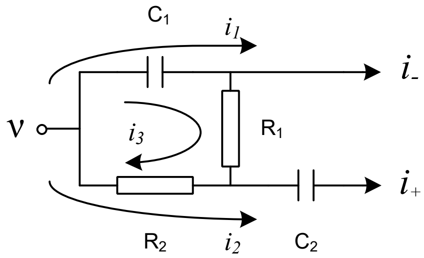
$$j\omega R_1 C_2 = j\omega R_1 C_1 + j\omega R_2 C_2$$

$$R_1 C_2 = R_1 C_1 + R_2 C_2 = 1$$

$$\frac{R_2}{R_1} + \frac{C_1}{C_2} = 1$$

Circuit 2




Mesh currents

Circuit 2: Impedance matrix

$$\Delta z = \begin{vmatrix} \frac{1}{sC_1} & 0 & \frac{1}{sC_1} \\ 0 & R_2 + \frac{1}{sC_2} & -R_2 \\ \frac{1}{sC_1} & -R_2 & R_1 + R_2 + \frac{1}{sC_1} \end{vmatrix}$$

$$\Delta z \cdot i_1 = \begin{vmatrix} v & 0 & \frac{1}{sC_1} \\ v & R_2 + \frac{1}{sC_2} & -R_2 \\ 0 & -R_2 & R_1 + R_2 + \frac{1}{sC_1} \end{vmatrix}$$

$$(\Delta z \cdot i_1)/v = \Delta_{11} + \Delta_{21}$$

$$= \begin{vmatrix} R_2 + \frac{1}{sC_2} & -R_2 \\ -R_2 & R_1 + R_2 + \frac{1}{sC_1} \end{vmatrix} - \begin{vmatrix} 0 & \frac{1}{sC_1} \\ -R_2 & R_1 + R_2 + \frac{1}{sC_1} \end{vmatrix}$$

$$= R_1 R_2 + R_2^{\ 2} + \frac{R_2}{sC_1} + \frac{R_1}{sC_2} + \frac{R_2}{sC_2} + \frac{1}{s^2 C_1 C_2} - R_2^{\ 2} - \frac{R_2}{sC_1}$$

$$= R_1 R_2 + \frac{R_1}{sC_2} + \frac{R_2}{sC_2} + \frac{1}{s^2 C_1 C_2}$$

$$\Delta z \cdot i_2 = \begin{vmatrix} \frac{1}{sC_1} & v & \frac{1}{sC_1} \\ 0 & v & -R_2 \\ \frac{1}{sC_1} & 0 & R_1 + R_2 + \frac{1}{sC_1} \end{vmatrix}$$

$(\Delta z \cdot i_2)/v = \Delta_{12} + \Delta_{22}$

$$= \begin{vmatrix} 0 & -R_2 \\ \frac{1}{sC_1} & R_1 + R_2 + \frac{1}{sC_1} \end{vmatrix} - \begin{vmatrix} \frac{1}{sC_1} & \frac{1}{sC_1} \\ \frac{1}{sC_1} & R_1 + R_2 + \frac{1}{sC_1} \end{vmatrix}$$

$$= -\frac{R_2}{sC_1} + \frac{R_1}{sC_1} + \frac{R_2}{sC_1} + \frac{1}{s^2 C_1^{\ 2}} - \frac{1}{s^2 C_1^{\ 2}}$$

$$= \frac{R_1}{sC_1}$$

For sinusoidal signal:

$$s^2 = (j\omega)^2 = -\omega^2$$

$$\frac{1}{s^2 C_1 C_2} = -\frac{1}{\omega^2 C_1 C_2}$$

For stable sinusoidal oscillation

$$i_1 = i_2$$

Therefore:

$[(\Delta z \cdot i_1)/v] = [(\Delta z \cdot i_2)/v]$

Equating real and imaginary parts of $i_1$ and $i_2$:

**Real** (gives the frequency of oscillation):

$$R_1 R_2 - \frac{1}{\omega^2 C_1 C_2} = 0$$

$$\omega^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

$$\omega = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \text{ r/s}$$

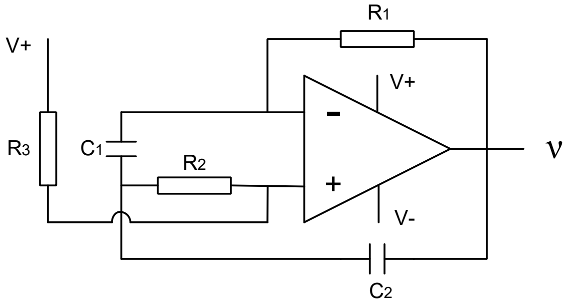
$$f = \frac{\omega}{2\pi} \text{ Hertz}$$

**Imaginary** (gives the condition for oscillation)

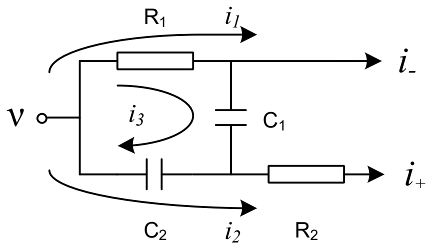
$$\frac{R_1}{C_2} + \frac{R_2}{C_2} = \frac{R_1}{C_1}$$

$$\frac{C_2}{C_1} = 1 + \frac{R_2}{R_1}$$

Circuit 3




Circuit 3 Mesh currents




Circuit 3: Impedance matrix

$$\Delta z = \begin{vmatrix} R_1 & 0 & R_1 \\ 0 & R_2 + \frac{1}{sC_1} & -\frac{1}{sC_2} \\ R_1 & -\frac{1}{sC_2} & R_1 + \frac{1}{sC_1} + \frac{1}{sC_2} \end{vmatrix}$$

$$\Delta z \cdot i_1 = \begin{vmatrix} v & 0 & R_1 \\ v & R_2 + \frac{1}{sC_2} & -\frac{1}{sC_2} \\ 0 & -\frac{1}{sC_2} & R_1 + \frac{1}{sC_1C_2} \end{vmatrix}$$

$$(\Delta z \cdot i_1)/v = \Delta_{11} + \Delta_{21}$$

$$= \begin{vmatrix} R_2 + \frac{1}{sC_2} & -\frac{1}{sC_2} \\ -\frac{1}{sC_2} & R_1 + \frac{1}{sC_1} + \frac{1}{sC_2} \end{vmatrix} - \begin{vmatrix} 0 & R1 \\ -\frac{1}{sC_2} & R_1 + \frac{1}{sC_1} + \frac{1}{sC_2} \end{vmatrix}$$

$$= R_1R_2 + \frac{R_2}{sC_1} + \frac{R_2}{sC_2} + \frac{R_1}{sC_2} + \frac{1}{s^2C_1C_2} - \frac{1}{s^2{C_2}^2} - \frac{R1}{sC_2}$$

$$= R_1R_2 + \frac{R_2}{sC_1} + \frac{R_2}{sC_2} + \frac{1}{s^2C_1C_2}$$

$(\Delta z \cdot i_2)/v = \Delta_{12} + \Delta_{22}$

$$= \begin{vmatrix} 0 & -\frac{1}{sC_2} \\ R_1 - \frac{1}{sC_2} & R_1 + \frac{1}{sC_1} + \frac{1}{sC_2} \end{vmatrix} - \begin{vmatrix} R_1 & R_1 \\ R_1 & R_1 + \frac{1}{sC_1} + \frac{1}{sC_2} \end{vmatrix}$$

$$= -\frac{R_1}{sC_2} + R_1^2 + \frac{R_1}{sC_1} + \frac{R_1}{sC_2} - R_1^2$$

$$= \frac{R_1}{sC_1}$$

For sinusoidal signal:

$$s^2 = (j\omega)^2 = -\omega^2$$
$$\frac{1}{s^2 C_1 C_2} = -\frac{1}{\omega^2 C_1 C_2}$$

For stable sinusoidal oscillation

$$i_1 = i_2$$

Therefore:

$[(\Delta z \cdot i_1)/v] = [(\Delta z \cdot i_2)/v]$

Equating real and imaginary parts:

**Real** (gives the frequency of oscillation):

$$R_1 R_2 - \frac{1}{\omega^2 C_1 C_2} = 0$$
$$R_1 R_2 = \frac{1}{\omega^2 C_1 C_2}$$

$$\omega^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

$$\omega = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \text{ r/s}$$

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**Imaginary** (gives the condition for oscillation)

$$\frac{R_1}{C_1} = \frac{R_2}{C_1} + \frac{R_2}{C_2}$$

$$\frac{R_1}{R_2} = 1 + \frac{C_1}{C_2}$$

*Reference:* Schaum's Outline Series
ELECTRIC CIRCUITS
By Joseph A. Edminister